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# ROYAL AIRCRAFT ESTABLISHMENT

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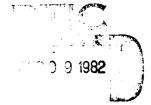
February 1982

# THE VALUE OF PHOTOGRAPHIC OBSERVATIONS IN IMPROVING THE ACCURACY OF SATELLITE ORBITS

by

D. G. King-Hele

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Procurement Executive, Ministry of Defence Farnborough, Hants

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### 17. Abstract

With the great majority of close Earth satellites (99.5%), orbit determinations normally have to rely on radar and visual observations of accuracy about 200 m. When there are also camera observations, of accuracy about 10 m if the height is about 1000 km, the orbital accuracy can be considerably improved; but the degree of improvement has been difficult to predict.

Large numbers of observations from the Russian AFU-75 cameras in the years 1971-3 have recently become available, particularly of the balloon-satellite Explorer 19, from the observing stations at Riga, Helwan, Khartoum and Fort Lamy. These, together with observations from the Hewitt camera at Edinburgh, have been added to improve three orbits of Explorer 19 in October 1972 already well determined from radar, visual and kinetheodolite observations. The addition of the camera observations improves the accuracy of some of the orbital elements by factors of up to 11, while other orbital elements benefit much less. The reasons for the differences are identified, the most important being the number of camera plates and the spread of the stations in latitude. As a by-product of the work, the cross-track residuals of 12 visual observers were evaluated: their mean was 0.02°.

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# SUMMARY

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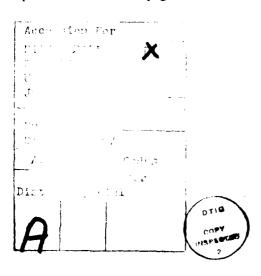
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#### INTRODUCTION

About 0.5% of the satellites in orbit can be tracked with an accuracy of 1 m or better, because they carry radio transmitters for Doppler or range-and-range-rate measurements, or are fitted with corner reflectors for laser tracking. Very few of this privileged minority of satellites are of interest for upper-atmosphere researches, since nearly all are in orbits high enough to be little affected by the atmosphere. So the objects useful for upper-atmosphere studies are to be found among the 99.5% of unprivileged satellites, for which the available observations are much less accurate, ranging from visual and radar observations, of about 200 m accuracy, to observations by satellite cameras such as the AFU-75, Baker-Nunn and Hewitt cameras, which are accurate to about 10 m if the satellite is at a height of about 1000 km.

Recently, large numbers of observations from the Russian AFU-75 cameras have become available, covering the years 1971-3. The observations, made during the 'Atmosphere' programme, are mainly of three satellites, Explorer 19 (1963-3A) and Explorer 39 (1968-66A), which were balloons, and Polyot 1 (1963-43A), which is a satellite of normal construction. Observations of these satellites between 1971 and 1973 are also available in good numbers from the Hewitt cameras at Malvern and Edinburgh, from the kinetheodolite at the South African Astronomical Observatory, from visual and theodolite observers and from the US Navspasur system (though unfortunately US Navy observations of 1963-43A were not requested until 1976).

It has long been accepted as a truism that more accurate observations will lead to more accurate orbits. But the advantage of adding photographic observations has proved to be very variable and unpredictable, depending on the number, distribution and accuracy of other observations, the orbital geometry, the severity of drag and many other factors.

The AFU-75 observations have previously been assessed by Hiller<sup>1</sup>, who determined orbits in which the AFU observations were dominant: this work indicated the accuracy of the observations. The aim of the present paper is different - to assess the advantages of adding camera observations from the AFU-75 or Hewitt cameras, or both, to orbits which are already reliably determined from visual, radar and kinetheodolite observations well distributed in latitude.

The first problem was to find a month when one of the three satellites mentioned earlier was extensively observed by both the AFU-75 cameras and the Hewitt cameras. Also, in order to avoid orbital bias, the month had to be chosen at a time when the satellite was visible from the southern hemisphere and plenty of observations from the kinetheodolite at the South African Astronomical Observatory were available. Explorer 19 was the chosen satellite and its orbit was calculated at three epochs in October 1972 when the observational coverage was excellent. The only objection to this choice is that Explorer 19 was a balloon satellite and was much affected by solar radiation pressure, which is not included in the orbital model of PROP6, the orbit refinement program used in determining the orbits; so it is possible that the final orbits, obtained using both sets of camera observations, may not be quite as accurate as the observations warrant.

#### 2 THE AFU-75 CAMERAS

The Russian AFU-75 camera was designed in 1965 at the tracking station of Riga University by K.K. Lapushka and M.K. Abele. The camera has a four-axis mounting, stands on a specially designed equatorial platform, and can be rotated to follow the satellite. The optical system is a seven-lens arrangement of aperture 21 cm and focal length 74 cm. The field of view is  $10^{\circ} \times 14^{\circ}$  and the images are recorded on film 19 cm wide. There is also a guiding telescope of aperture 12 cm for visual control of the tracking if necessary. Timing is by crystal clock, oscillograph and radio receiver. Satellites down to magnitude 10 can be recorded. A full description of the AFU-75 has been given by A.G. Massevitch and A.M. Losinsky  $^3$ . Fig 1 gives a photograph of the camera.

The AFU-75 cameras began operating about 1970 at the following sites: Riga, Uzhgorod, Zvenigorod and Pulkovo in the USSR; Ondrejov (Czechoslovakia); Sofia (Bulgaria); Baja (Hungary); Ulan-Bator (Mongolia); Havana (Cuba); Helwan, near Cairo (Egypt); Afghoi (Somalia); the Kerguelen Islands (Indian Ocean); and Mirny in the Antarctic. The locations of the cameras have been changed from time to time, but large numbers of observations continue to be made. An accuracy between 1 and 3 seconds of arc is usually quoted, and an independent study by Hiller , of orbits determined mainly from AFU-75 observations, suggests that 3 seconds of arc is frequently achieved, though it is wiser to assume 10 seconds because of possible misinterpretations of station coordinates, reference frames, timing systems, etc.

# 3 THE THREE ORBITS

In choosing the dates for the three orbit determinations of Explorer 19 in October 1972, the aim was to ensure that observations were available from all three of the following sources: (1) a Hewitt camera; (2) an AFU-75 camera; and (3) the kinetheodolite at the South African Astronomical Observatory. Visual and US Navy observations were plentiful throughout the month. The time intervals chosen are given in Table 1 opposite, together with the total numbers of observations from each source used in the final orbit and the latitudes of the stations. The total numbers of observations available were much greater, but the maximum number in a normal PROP run is 100, and this limit was retained so that the comparisons would be between normal cuns. A maximum of six observations from one station on one transit was set, to avoid overweighting of the observations from a single station.

The epoch of an orbit determination with PROP is always at midnight, and the epochs in Table 1 are chosen to be as near as possible to the 'centre of gravity' of the most accurate observations.

The results for the three orbits are described in sections 4, 5 and 6.

Number of observations from each source in each of the three orbit determinations

0.1.1	,	2	2
Orbit	•	2	د
Epoch, 1972	Oct 3.0	Oct 13.0	Oct 24.0
Times of observations	Oct 1.8-5.9	Oct 10.7-15.9	Oct 21.6-26.0
Edinburgh Hewitt camera, 56°N	8	5	5
Riga, 57°N	6	0	0
AFU-75) Helwan, 30°N	0	19	20
camera Fort Lamy, 12°N	0	0	7
Khartoum, 15°N	0	0	3
Cape kinetheodolite, 34°S	4	2	8
US Navy, 33°N	20	29	29
Visual, mainly 50-56°N, (also 21°N, 18°S & 35°S	56	40	13
Jokioinen, Finland, 61°N	6	4	2
Malvern radar, 52°N	0	0	13

# 4 ORBIT 1, EPOCH 1972 OCTOBER 3.0

#### 4.1 Results

In the PROP orbital model, the mean anomaly  $\,\mathrm{M}\,$  is expressed as a polynomial in time  $\,\mathrm{t}\,$  after epoch,

$$M = M_0 + M_1 t + M_2 t^2 + M_3 t^3 + \dots,$$
 (1)

and for orbit 1 it was found that the best fit was obtained by stopping at the  $M_3$  coefficient.

The other orbital parameters are the inclination to the equator i and the right ascension of the ascending node  $\Omega$ , which together specify the orientation of the orbital plane; and the eccentricity e and argument of perigee  $\omega$ , which specify the shape of the orbit and its orientation within the orbital plane. In the PROP model each of these four parameters is usually expressed as a linear function of time, and it is customary to determine values of e, i,  $\Omega$  and  $\omega$  at epoch by fitting the observations, while accepting as correct the rates of change of e, i,  $\Omega$  and  $\omega$  calculated theoretically within the PROP model.

Explorer 19, however, was a balloon of diameter 3.6 m and mass 7 kg, and it is possible that the actual rates of change of the orbital elements may be sufficiently affected by solar radiation pressure to depart considerably from the values calculated within PROP, which ignore solar radiation pressure. The actual mean values for the rates of change of i,  $\Omega$  and  $\omega$  between orbits 1 and 2 (or 2 and 3) do not differ significantly from the values assumed within PROP, but the variation of e is very different: the mean value of  $10^3$ ė between orbits 1 and 2 is 0.014 per day, whereas

a value of -0.026 per day is expected in the absence of solar radiation pressure. The orbits were therefore re-run with  $\dot{e}$  free, but it was found that only orbit 1 needed this extra parameter in the model. There was no improvement in the fitting for orbits 2 and 3, presumably because the mean value of  $10^3 \dot{e}$  between orbits 2 and 3 is -0.024, which is close to the value expected in the absence of solar radiation pressure, namely -0.032. So, for orbits 2 and 3,  $\dot{e}$  was not included as a parameter to be determined. For orbit 1 the value obtained was  $10^3 \dot{e} = 0.077 \pm 0.008$ .

Apart from this modification, the orbit determination was straightforward, though a number of the original observations had to be omitted to keep the numbers down to 100. The final orbit obtained, using all the observations of Table 1, is given in the first row of Table 2. Next the orbit was run with the Hewitt camera (HC) observations included,

Table 2

Orbit 1, epoch 1972 October 3.0: orbital elements and standard deviations

	đ	e	i	a	J.	М <sub>О</sub>	. <b>M</b> .1	м <sub>2</sub>	м <sub>3</sub>	ε	N
Final orbit (all obser- vations)	7686.9864 11	0.066386	78.8574 9	161.7846 12	48.911	0.629	4638.5312 10	0.0388	-0.0006 1	0.54	100(0)
With HC (but not AFU)	7686.9840 12	0.066404 15	78,8590 10	161.7848 13	48.928 7	0.611	4638,5333 11	0.0393	-0.0009	0.53	94(1)
With AFU (but not HC)	7686.9824 16	0.066410 16	78,8577 9	161.7838 12	48.937 8	0.598	4638,5348 14	0.0420	-0.0018	0.51	92(1)
Basic (visual, US Navy, CK)	7686.9825 18	0.066395 19	78.8589 11	161.7848 14	48.939	0.595	4638.5338 17	0.0440	-0.0023 5	0.52	86(1)

Key:

- a semi major axis (km)
- e eccentricity i inclination (deg)
- n right ascension of node (deg)
- w argument of perigee (deg)

 $M_0$  mean anomaly at epoch (deg)

M. mean motion n (deg/day)

M2 & M3 additional coefficients in polynomial for M,

equation (1)

measure of fit
number of observations used (number rejected

in brackets)

but with the AFU-75 omitted; then with the AFU observations included but the HC omitted; the fourth row gives the basic orbit obtained when all the camera observations are omitted, that is the orbit determined from just the US Navy, visual and Cape kinetheodolite (CK) observations.

#### 4.2 Discussion

The orbits in Table 2 are best looked at in reverse order, starting with the basic orbit and noting the improvements on adding either the AFU observations or the HC, and then both.

The accuracy of the basic orbit in Table 2 is equivalent to 150 m radially and across track, and, in view of the fact that there are 86 well-distributed observations, this accuracy is not particularly good: for example. Walker 4 obtained an average radial

standard deviation of 100 m on the 61 orbits of 1971-106A without Hewitt camera observations, relying on US Navy, visual and kinetheodolite observations, as here.

The AFU-75 camera observations were from Riga: all six were on one transit and they were given accuracies of 10 seconds of arc. Table 2 shows that the inclusion of these AFU observations (but not the HC) reduces the standard deviations of the orbital elements by between 10% and 30%, the average being 20%. The coordinates of the Riga camera were taken from Ref 5, and no difficulties arose.

The inclusion of the HC observations (instead of the AFU) has a rather similar effect on the standard deviations of the geometrical elements e, i, i. and  $\omega$ . But the inclusion of the Hewitt camera observations greatly improves the standard deviations of  $M_1$ ,  $M_2$  and  $M_3$ , particularly  $M_2$ , for which the accuracy is improved by a factor of 5. This is because there are two transits recorded by the Hewitt camera on successive days. The second group, on October 2, were given an accuracy of 3 seconds of arc, but the group on October 1 had to be degraded to 20 seconds of arc to avoid rejection. This may be because the PROP model cannot cope with the effects of solar radiation pressure.

The final orbit, with both HC and AFU, shows further small improvements in most of the standard deviations; but for  $\omega$  and  $M_{\cap}$  the standard deviations are halved.

Overall, the standard deviations are reduced between the basic and the final orbit by factors of between 1.2 and 8. The improvement is least for inclination and right ascension, and would probably have been much greater if the cameras had been separated in latitude. As it is, the Edinburgh camera is at latitude  $55.7^{\circ}$ N and the Riga camera is at latitude  $56.9^{\circ}$ N, so that their geometrical effects would be expected to be - and are - very similar.

In the past the standard deviations from PROP have proved quite reliable, for a wide variety of orbits, so there is every reason to believe that the improvements in accuracy indicated above are measured realistically. It is worth noting too that the four values of eccentricity in Table 2, and those of inclination and right ascension, are all consistent to within the sum of their standard deviations. The same applies to  $\omega + M_0$ , though not to  $\omega$  and  $M_0$  separately. There are some discrepancies among the values of  $M_0$ - $M_3$ ; this is to be expected when fitting a cubic.

# 5 ORBIT 2, EPOCH 1972 OCTOBER 13.0

# 5.1 Results

The results for orbit 2 are presented in a similar format in Table 3. With this orbit there are two parameters fewer to be determined than with orbit 1, because  $M_3$  is absent and  $\dot{\mathbf{e}}$  is not determined.

For orbit 2, there was only one transit from the Hewitt camera, on October 12, while the AFU-75 observations all came from the camera at Helwan in Egypt, but were from four transits on successive days, October 11, 12, 13 and 14. The coordinates of Helwan were those given by Marsh, Douglas and Klosko<sup>5</sup>, and the assumed accuracy was 10 seconds of arc.

Table 3
Orbit 2, epoch 1972 October 13.0: orbital elements and standard deviations

	3	ę	i		-	М	v	u .		
Final orbit (all obser- vations)	7686,2967 2	0.066523	78,8523 6	151.6866	28.311 6	308.5e.	.n34,155h	. 2		
With HC (but not AFU)	7686.2996 9	0.066513	78.8568 11	151.6808 14	28.307 9	308,571   9	-634.1531 - 634.1531 - 6	!		
With AFU (but not HC)	7686.2963 2	0.066523			28.314	308,562	4639.1361 2	0.0218  -	3.56	! 44 0) :
Basic (visual, US Navy, CK)	7686.2998	0.066528					40.34.1326 3		0.67	75+17 1

For key, see Table 2

The orbit computations proceeded smoothly. It was sometimes difficult to decide which observations to omit, so as to reduce the numbers to 100, but the choice is probably not crucial.

It was not possible to use the AFU-75 observations recorded at Fort Lamy (Station 1911, see section 6.1) on October 13, even after several experiments in changing the time of the observations.

#### 5.2 Discussion

The accuracy of the basic orbit in Table 3 is about the same as that of the basic orbit 1, except that  $\rm M_1$  and  $\rm M_2$  are better determined, as would be expected in the absence of  $\rm M_3$ .

The addition of the AFU observations greatly improves the standard deviations of  $M_1$  and  $M_2$ , by factors of 4 and 3; this is because the AFU observations are well spread in time. There are also impressive improvements in other parameters, the standard deviations of  $\Omega$  and  $\omega$  both being halved. The improvement in the accuracy of the inclination is less, 25%, probably because the observations are nearer the equator than the apex. The improvement in the standard deviation for the eccentricity is only 10%, probably because the latitude of the camera  $(30^{\circ}\text{N})$  is similar to that of the US Navy observations  $(33^{\circ}\text{N})$ . The strongest determinants of the eccentricity are observations from the kinetheodolite in the southern hemisphere, and these are already included in the basic orbit.

The addition of the HC observations (without the AFU) has a rather different effect, because all the observations are on a single transit and consequently there is very little improvement in the accuracy of  $M_1$  and  $M_2$ . Also, since the Hewitt camera is at high latitude, the improvement in the accuracy of the right ascension is only 10%. But the effect on eccentricity and inclination is just as great as is produced by the much larger number of AFU observations, the higher latitude being more favourable to orbital accuracy.

The final orbit in Table 3 combines the best of both worlds, as it were, and we see the great advantage of having two cameras at widely differing latitudes. As compared with the basic orbit, the standard deviations of i,  $\Omega$ ,  $\omega$  and  $M_2$  in the final orbit are all reduced by a factor of between 2 and 3, while  $M_1$  (and hence a) is improved by a factor of 4. The one stubborn parameter is the eccentricity, for which the standard deviation is reduced by only 20%: a greater improvement would be expected if one of the cameras had been in the southern hemisphere.

# 6 ORBIT 3, EPOCH 1972 OCTOBER 24.0

# 6.1 Results

Table 4 records the results for orbit 3, with the same format as Table 3, apart from an additional entry which will be discussed later. Again there was only one transit from the Hewitt camera, on October 25, but the AFU-75 observations were more numerous and more varied, with 30 observations from three stations, Fort Lamy, Helwan and Khartoum, covering all the five days of the orbit determination, October 21-25.

Table 4
Orbit 3, epoch 1972 October 24.0: orbital elements and standard deviation

	a	e	i	.3	پ	<b>M</b> <sub>0</sub>	M	м <sub>2</sub>	٤	;
Final orbit (all obser- vations)	7685.7218 1	0.066261	78.8501 }	140.5627	5.506 5	221.381	4639.6762 1	0.0284	0.58	100(0)
With HC (but not AFU)	7685.7237 4	0.066283 7	78.8518 4	140.5600	5.541	221.349 6	4639.6745	0.0272	0.44	70(0)
With AFU (but not HC)	7685.7219	0.066260 4	78.8516 4	140.5622	5.511	221.376	4639.6761	0.0283	0.57	95(0)
Basic (visual, US Navy, CK)	7685.7249 6	0.066270 8	78.8510 11	140.5597	5.540	221.351	4639.6734	0.0266	0.44	65(0)
Final (USSR coordinates)	7685.7226 1	0.066283	78.8522 !	140.5591 1	5.531 5	221.356	4639.6755	0.0276	0.54	100(2)

For key, see Table 2

At the outset the only station coordinates available for Fort Lamy were: "longitude 15<sup>0</sup>02'E, latitude 12<sup>0</sup>07'N". So it was necessary to adjust the coordinates to fit the observations. After consulting maps and noting that the height of Fort Lamy airport is recorded as 295 m, the height was taken as 300 m; the latitude and longitude were then adjusted by differential correction until four observations on October 21 and three on October 23 were accepted.

For Khartoum, two corrections were needed: I minute had to be added to the times of all the observations; and the station coordinates had to be adjusted. To allow for

the uncertainties in the station coordinates, the observational accuracies were taken as 20 seconds of arc for both Fort Lamy and Khartoum.

The station coordinates used, relative to a spheroid of semi major axis 6378.155 km and semi minor axis 6356.770 km, were as follows:

Fort Lamy: longitude 15.0636°E; latitude 12.1050°N; height 300 m Khartoum: longitude 32.6607°E; latitude 15.4397°N; height 350 m.

Since the rms residual for Fort Lamy was 0.2 minute of arc and the average range was about 2000 km, the error in the station coordinates is likely to be of order 100 m.

After the 'final orbit' in the top row of Table 4 had been determined,

Dr S.K. Tatevian, of the Astronomical Council of the USSR Academy of Sciences, sent me the coordinates of the African tracking stations as determined, in the Smithsonian Standard Earth III system, by Tawadrus<sup>6</sup>. These 'USSR' coordinates differ from those used in the 'final orbit' by more than 100 m, the values being as follows:

Station		X (km)	Y (km)	Z (km)
1901 Helwan	'final orbit'	4728.089	2879.692	3157.237
	USSR	4728.422	2879.597	3156.715
1904 Khartoum	'final orbit'	5177.376	3318.804	1687.145
	USSR	5177.196	3316.980	1691.489
1911 Fort Lamy	'final orbit'	6023.213	1621.083	1328.828
	USSR	6023.457	1620.775	1328.387

The definitions of X, Y and Z are given in the PROP manual<sup>7</sup>.

To provide a comparison, the 'final orbit' was re-run with the USSR coordinates, and this led to the orbit given in the last row of Table 4. (Note that it was necessary to subtract 0.75 second from the times of the Khartoum observations, in order to avoid rejection; it was then possible to reduce their a priori error to 10 seconds of arc.)

The orbit in the last row of Table 4 has standard deviations identical to those of the 'final orbit', but the values of some parameters change slightly, as would be expected with the change in coordinates. The large change in the Khartoum Z coordinate may seem curious, but it presumably arises because the 'adjusted' coordinates used in the 'final orbit' were adjusted to take out the 0.75 second timing error.

# 6.2 Discussion

Table 4 shows that the basic orbit is rather more accurate than with orbits 1 and 2: the standard deviations in eccentricity and inclination correspond to radial and cross-track errors of 60 and 140 m.

When the observations from the three AFU-75 cameras are added, there is a great improvement in accuracy, the standard deviations of  $\rm M_1$  and  $\rm M_2$  being reduced by factors of 5, that of  $\Omega$  by a factor of 4, that of i by a factor of 3, and that of e by a factor of 2. This shows that great advantage of having photographic observations from different locations, even though all three stations are in North Africa.

When the Hewitt camera observations are added (without the AFU) there is a modest improvement, of 10-30%, in all the standard deviations except that of the inclination, for which the standard deviation is reduced by a factor of 3. Again the high latitude of the Hewitt camera proves most effective in sharpening the accuracy of the inclination.

In the final orbit, with all the observations, we see a multiplication of the good effects: the improvement over the orbit with AFU (but not HC) is 10-30%, except for the inclination, where the improvement is by a factor of 4. Thus, relative to the basic orbit, the standard deviations on the final orbit are 11 times better for inclination, 7 times better for  $M_1$  and  $M_2$ , 6 times better for  $M_1$ , and twice as good for eccentricity; the reduction for  $M_1$  and  $M_2$ , is only 30%. Again it would be expected that the standard deviations in e,  $M_1$  and  $M_2$  would be greatly improved if camera observations from the southern hemisphere had been available.

The 'final' orbit using the USSR coordinates, in the last row of Table 4, is probably more reliable than the 'final orbit' in the first row, (a) because the station coordinates are independently determined, and (b) because this orbit is so close to the orbit 'With HC (but not AFU)', the differences being less than the sum of the standard deviation, except for M<sub>1</sub> (and the related parameter a). Since the radial and cross-track accuracies on the 'final' orbit are 30 m and 10 m respectively, and the accuracies of the camera observations are not likely to be better than 20 m at a range of 2000 km, the results are as good as could ever be expected.

# GENERAL CONCLUSIONS ON ORBITAL ACCURACY

The pattern emerging from the comparisons in sections 4 to 6 has a recognizable logic, which may be summarized as follows.

- (1) When an orbit has been reliably determined from well-distributed visual, radar and kinetheodolite observations, the addition of camera observations always improves the accuracy, but the degree of improvement depends crucially on the orbital geometry, the location of the cameras, and the number of transits observed.
- (2) If the camera observations are on only one transit from only one station, at a latitude already well represented by observations (as with the HC observations in orbits 2 and 3) the improvement in the standard deviations for most of the orbital parameters is likely to be only about 20%, but a greater improvement, by a factor of up to 3, can be expected in either the inclination (if the camera is at high latitude) or the right ascension (if the camera is at low latitude).
- (3) If the camera observations are on more than one transit, though still from one station only, there is a dramatic improvement, typically by a factor of 4, in the parameters defining the mean anomaly, particularly  $M_1$  and  $M_2$ ; and the geometrical parameters are improved in accuracy by a factor of up to 2, relative to the basic orbit.
- (4) If the camera observations are from stations at different latitudes, the orbital accuracy is very greatly improved, by factors of between 6 and 11 for most parameters in orbit 3. The exceptions are the eccentricity and argument of perigee, for which the factor of improvement is about 2: the reason for this relatively poor performance is

that even in orbit 3 the distribution of the camera observations is still somewhat limited, all of them being from latitudes between  $56^{\circ}N$  and  $12^{\circ}N$  on south-going transits, that is, over only 12% of the orbit. This leads to the next conclusion.

- (5) Improvements in the accuracy of e and to match those in the other parameters namely, factors of order 10 call for ob rvations very widely separated in latitude. In practice, this means that at least one camera needs to be well to the south of the Equator. A separation of about  $90^{\circ}$  in latitude is likely to be optimum, and this separation is nearly achieved by the Malvern Hewitt camera (latitude  $52^{\circ}$ N) and the Hewitt camera newly installed at Siding Spring in Australia (latitude  $32^{\circ}$ S).
- (6) Finally, it should be remembered that the orbital model in PROP does not allow for the effects of solar radiation pressure; so the final orbits obtained here (from all observations) are probably not as accurate as the observations would allow if the orbital model were perfect. In other words, the camera observations have not received full justice their power to improve accuracy may be greater than is displayed here.

#### 8 ANALYSIS OF OBSERVATIONS

#### 8.1 Rms residuals

Part of the output from PROP consists of the residuals of each observation in right ascension and declination, that is, the value given by the observer minus the value on the final orbit at the time specified by the observer. The residuals thus record the total of observational error (in both position and time) and error in the orbit. The residuals have been printed out using the ORES program and their rms values are recorded in Table 5, for all stations with 3 or more observations.

Table 5 calls for several comments. First, the rms is a really misleading statistic when the total numbers of observations are so small. For example, with station 1, if the largest of the 8 residuals is omitted, the right ascension rms is reduced from 2.6 to 1.4 minutes of arc, and the declination rms is reduced from 3.2 to 2.1 minutes of arc. The overall rms then becomes 2.5 minutes of arc, the same as for station 2 (which is of the same type). By contrast, the value for station 5 is misleadingly low, but based on only 4 observations. However, it should be remembered that the visual observations omitted in the course of the runs were usually those with high residuals, so the rms values for most visual observers are probably quite realistic.

The residuals for the Hewitt camera have a particularly misleading rms value, the total of 0.09 minute of arc being much larger than is usual for that camera. For example, with the satellite 1971-106A, Walker obtained a total rms residual of 0.04 minute of arc from 51 observations from station 2534, as compared with 0.09 minute of arc in Table 5. However, if the observations on October 1 are omitted, the right ascension and declination residuals for station 2534 in Table 5 are both reduced to 0.02 minute of arc, giving a total rms of 0.03 minute of arc from 15 observations, instead of 0.09 minute of arc from 18 observations, as shown in Table 5 by the second set of values, in square brackets. This nicely displays the fragility of the rms. Since the poor fitting of the observations on October 1 can probably be attributed to the PROP model not accurately

representing the orbit of Explorer 19 over more than a day or two, the residuals in square brackets are to be preferred as an indicator of the accuracy - 2 seconds of arc.

Table 5

Rms residuals for stations with three or more observations

Station		Number of	Rms residuals				
No.	Name	observations	Range (km)	RA mi	Dec nutes of	Total arc	
1	US Navy	8		2.6	3.2	4.1	
2	US Navy	6		0.9	2.3	2.5	
4	US Navy	3		2.1	1.3	2.5	
5	US Navy	4		1.0	0.6	1.2	
6	US Navy	7		0.9	1.4	1.6	
29	US Navy	48	1.2	0.4	0.8		
1084	Riga (AFU)	6		0.08	0.14	0.16	
1901	Helwan (AFU)	36		0.23	0.18	0.29	
1904	Khartoum (AFU)	3		0.10	0.17	0.20	
1911	Fort Lamy (AFU)	7		0.13	0.13	0.19	
1963	Jokioinen (theod.)	12		1.5	3.0	3.3	
2265	Farnham (vis)	5		2.4	1.0	2.6	
2304	Malvern radar	13	0.9	2,9	4.3		
2392	Cowbeech (vis)	6		1.8	1.7	2.5	
2414	Bournemouth (vis)	27		2.9	2.3	3.7	
2420	Willowbrae (vis)	9		1.0	1.4	1.7	
2421	Malvern 4 (vis)	17		1.0	1.7	1.9	
2424	Stratford-on-Avon (vis)	3	{	1,1	2.0	2.3	
2513	Colchester (vis)	4		3.3	3.2	4.6	
2534	Earlypoint (HC)	{ 18 [15		0.05	0.08 0.02	0.09 0.03]}	
2550	Masira (vis)	4	[	1.9	1.4	2.3	
2577	Cape kinetheodolite	14		0.9	1.3	1.6	
2588	Antarl (Adelaide) (vis)	5		2.0	2.0	2.8	
4126	Groningen (vis)	3		1.8	2.5	3.1	
4130	Denekamp (vis)	7		3,6	2.3	4.3	
4159	Achel 2 (vis)	3		3.7	2.9	4.7	

The residuals for Riga, on October 3, may also be affected by the limitations of the PROP model and the only conclusion to be drawn is that the accuracy of the Riga AFU-75 camera is better than the total rms in Table 5, namely 9 seconds of arc.

The observations from the AFU-75 camera at Helwan (station 1901) are for nine transits, so larger residuals are to be expected because of the deficiencies in the PROP

model. This expectation is confirmed: the total rms of 36 observations from station 1901 is 0.29 minute of arc, twice that of station 1084. (These are the residuals when the USSR coordinates are used in orbit 3: the rms residual is 0.31 minute of arc from 39 observations when the coordinates of Ref 5 are used.)

The residuals for Khartoum and Fort Lamy are also those obtained using the USSR coordinates, and they are similar to those of Riga. (A much larger rms for Khartoum was recorded using the empirically adjusted coordinates.)

The residuals for the AFU-75 cameras are too much influenced by the limitations of the orbital model, and possible errors in station coordinates or reference systems, to allow an exact evaluation of their inherent accuracy. The fact that the AFU-75 observations improve the orbit almost as much as the Hewitt-camera observations is sufficient proof of their value and indicates that an inherent accuracy somewhere near 3 seconds of arc is probable.

The rms residuals of the other observations are slightly higher than usual. In their analyses of 1971-18B and 1971-106A, Hiller and Walker used 3050 observations from station 29 and obtained rms residuals of 0.6 km, 0.3 and 0.4 minutes of arc, compared with 1.2 km, 0.4 and 0.8 minutes of arc here. Again, this is probably attributable to the absence of solar radiation pressure perturbations in the orbital model. (The angular residuals for station 29 are geocentric and need to be multiplied by a factor of about 5 to make them comparable with the other angular residuals in Table 5.) Similarly, for the Cape kinetheodolite, Hiller obtained a total rms of 1.1 minutes of arc from 93 observations and Walker obtained 1.3 minutes of arc from 89 observations, as compared with 1.6 minutes of arc here.

#### 8.2 Rotated residuals

Visual observers often complain that the residuals in right ascension and declination, as printed in Table 5, mix up the cross-track and along-track errors. The along-track errors are likely to be dominant because they embody the timing errors as well as the along-track positional errors. If the timing error is of order 0.1 second, and the angular velocity of the satellite exceeds 0.2° per second, the along-track error due to timing will be 0.02° or more, while it is possible that the cross-track error may be as low as 0.01°.

The three orbits of Explorer 19 are ideal for testing this hypothesis, because the less accurate visual observations have mostly been eliminated while reducing the total numbers of observations to 100 for each orbit. In the PROP6 program<sup>2</sup>, Gooding provides for the printout of 'rotated residuals', which in normal circumstances are approximately along and cross-track. (For their exact definition, see Appendix A of Ref 2.) The three orbits have been re-run with the rotated-residual printout, and the cross-track errors for the visual observers are compared with the total errors in Table 6. In view of the small numbers, the arithmetic mean of the numerical values of the cross-track residuals has been given. Thus the first column of results may be regarded as giving the overall residuals and the second column as giving the cross-track error in the observations (the cross-track orbital error probably being negligibly small, especially at latitudes

50-60°N, close to that of the Hewitt camera). The results for the stations with only three observations should be treated with caution, since all of them are remnants of a larger number of observations from which the less accurate ones were omitted.

Table 6

Total and cross-track residuals for visual observers

	Station	Number of	Total rms	Mean cross-track	
No.	Name	observations	residual minutes of arc	residual minutes of arc	
2414	Bournemouth	27	3.7	1.7	
2421	Malvern 4	17	1.9	1.1	
2420	Willowbrae	9	1.7	0.9	
4130	Denekamp	7	4.3	2.6	
2392	Cowbeech	6	2.5	0.3	
2265	Farnham	5	2.6	1.4	
2588	Antarl	5	2.8	2.1	
2513	Colchester	4	4.6	1.6	
2550	Masira	4	2.3	1.4	
2424	Stratford-on-Avon	3	2.3	1.1	
4126	Groningen	3	3.1	0.4	
4159	Achel 2	3	4.7	1.3	

Table 6 clearly indicates that cross-track accuracies of 1 minute of arc are attainable by visual observers, and one observer, Gordon Taylor (2392), has achieved an average of 0.3 minute of arc, or  $0.005^{\circ}$ , with six observations. Russell Eberst (2420) has 0.9 minute of arc from nine observations, and David Brierley (2421) has 1.1 minutes of arc from 17 observations. Also it should be remembered that these observations have been chosen at random - a tiny sample of the 80000 so far made by Eberst and the 60000 so far made by David Hopkins (2414). If the observers had known they would be subjected to this critical analysis, they could probably have made even more accurate observations.

The obvious conclusion to be drawn is that visual observers should strive to achieve a cross-track accuracy of 0.01°: it is within their capabilities.

The rotated residuals for the other stations show a fairly random distribution between along- and cross-track values. This suggests that the along-track fitting of the photographic observations by the orbit is good, a not unexpected conclusion, because low values of  $\varepsilon$  could not be achieved if this fitting was poor.

#### 8.3 Conclusions from analysis of residuals

The total rms residuals (Table 5) are somewhat larger than usual for most stations, thus suggesting that the PROP model cannot represent the motion of Explorer 19 over about 5 days with an accuracy as good as that of the observations. For this reason the accuracy of the AFU-75 observations cannot be exactly evaluated, but there is every sign that these observations have inherent accuracies of about 3 seconds of arc.

Table 6 indicates that visual observers can often achieve cross-track accuracies of 1 minute of arc or better. So it is recommended that visual observers should aim for an

accuracy of 0.01°, though most observers will not often succeed in achieving this standard.

# 9 SUMMARY OF RESULTS

Each of the three orbits selected was differently influenced by the addition of Hewitt or AFU camera observations. The advantages of including camera observations can be very great: if the geometry is favourable, the accuracy of the orbital inclination can be improved by a factor of 10; but if the geometry is unfavourable and observations from the same latitude already exist, the improvements of standard deviations may be as small as 20%. The various possibilities are detailed in section 7.

The residuals of the observations tend to confirm that the AFU-75 observations have an inherent accuracy of about 3 seconds of arc. Rotated residuals for 12 visual stations indicate a mean cross-track error of 1.3 minutes of arc, and it is suggested that visual observers should aim for a positional accuracy of 0.01°.

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Fig 1 The AFU-75 camera (photograph by country of the Astronomical Council of the USSR Academy of Sciences)